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Exchanging Quantum Particles

Tomasz Bigaj
University of California, San Diego (USA)
University of Warsaw (Poland)

Résumé : La notion mathématique de permutation d'indices dans la description de l'état peut recevoir différentes interprétations physiques. Deux interprétations principales analysées dans cet article sont l'échange des essences et l'échange des heccéités. On soutient ici qu'adopter l'approche essentialiste conduit à la conclusion selon laquelle les particules quantiques d'un même type sont parfois discernables par leurs propriétés, conclusion contraire à la sagesse conventionnelle. Seule l'interprétation alternative de l'heccéité primitive permet de soutenir la thèse de l'indiscernabilité.

Abstract: The mathematical notion of a permutation of indices in the state description admits different physical interpretations. Two main interpretations analyzed in this paper are: exchange of essences and exchange of haecceities. It is argued that adopting the essentialist approach leads to the conclusion, contrary to the conventional wisdom, that quantum particles of the same type are sometimes discernible by their properties. The indiscernibility thesis can be supported only by the alternative interpretation in terms of primitive thisness.

1 Introduction

The philosophical debate over the meaning of the notion of individuality in quantum mechanics has been raging for decades. Given that virtually all positions and arguments have been thoroughly examined and re-examined, what rationale can be offered for yet another paper on such a well-researched topic? The main goal of this article is modest: I would like to revisit the notion of a permutation of objects which is a key but somewhat neglected

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concept. My suggestion is that if we acknowledge the fact that there is a conceptual gap between the unique mathematical notion of a permutation and its physical realizations, we can notice that there may be more than one acceptable interpretation of the latter. This is hardly a new and surprising idea; however to my knowledge no serious attempt to classify and examine possible interpretations of physical permutations of objects has been made in the context of quantum mechanics.

The starting point of this article is a discussion of four possible interpretations of the notion of physical exchange, of which I select two that seem best suited for an analysis of permutation invariance in quantum mechanics: exchange of essences and exchange of haecceities. A deeper investigation of both concepts reveals that they lead to radically different conclusions regarding the problem of discernibility of quantum particles. I argue that the essentialist interpretation of exchange invalidates the standard argument in favor of the Indiscernibility Thesis given by [French & Redhead 1988]. The proof of the Indiscernibility Thesis goes through only under the alternative, haecceitist interpretation. Moreover, I claim that essentialism actually strongly suggests that identical fermions and bosons can be absolutely discerned in some states by their quantum-mechanical properties. The formal proof of this fact which I present in the article is prefaced by a discussion of how to represent the properties of individual components of many-particle systems when meaningful operators are restricted to the symmetric ones. I end the article with an appeal for further study of the essentialist view, which is a relatively new and potentially fruitful approach.

2 Four notions of exchange

Formally, a permutation of an n-element set X is defined as a bijection mapping this set onto itself $\sigma : X \to X$. In the context of mathematical physics the most typical permutations are those of the indices in a compound mathematical object $\Psi(1, 2, \dots, n)$ (it can be a real-valued function, a vector, a density matrix, or any other object) representing a particular physical situation. Permutations applied to Ψ can be interpreted as mappings connecting it with objects arising as a result of permuting its indices: $\sigma(\Psi(1, 2, \dots, n)) = \Psi(\sigma(1), \sigma(2), \dots, \sigma(n))$. In the simplest case of an object containing just two indices the only non-trivial permutation leads from $\Psi(1, 2)$ to $\Psi(2, 1)$. The idea is of course that indices 1 and 2 are supposed to refer to physical entities (particles, properties, states of affairs, etc.), so the mathematical object $\Psi(2, 1)$ should correspond to the situation obtained from the one described by $\Psi(1, 2)$ by exchanging the required physical counterparts.

But the notion of swapping physical objects is not so clear-cut. We have to remember that mathematical concepts do not always perfectly match physical reality. Sometimes mathematical language creates artifacts (so-called surplus structures) not corresponding to anything real, and sometimes one physical situation receives many non-equivalent mathematical representations. In other cases one and the same mathematical concept can be interpreted physically in many different ways. Oftentimes failure to realize that there is no one-to-one correspondence between mathematical and physical concepts leads to serious misunderstandings.[1]

I suggest that at least four independent interpretations of the notion of exchange of physical objects can be given. The most natural way of thinking about exchanging physical objects is in terms of their location. If I asked you to swap this chair with that table, you would most probably move the chair to the place where the table stood, while simultaneously bringing the table to the location previously occupied by the chair. Hence the first interpretation presents itself naturally:

> *Exchange No 1* (exchange of locations). To exchange an object A located in r_A with an object B located in r_B is to create a situation in which in r_A there is an object which possesses all the non-relational (intrinsic) properties of B, while in r_B there is an object which possesses all the non-relational properties of A.

The second interpretation of exchange requires an introduction of an important notion of essential properties. As is standard in the literature, I will understand the essence of an individual object as the set of properties which this object possesses in all possible worlds in which it exists. My presupposition is that all objects have essences; however, I do not assume that each essence is unique. In fact, objects in the actual world can share their essences. One important example of essential properties is what quantum physicists call "intrinsic" (or state-independent) properties of elementary particles (among state-independent properties of an electron are its mass, charge, and total spin). It is of crucial importance to acknowledge that all particles of the same type (what physicists call, confusingly, "identical" particles, such as electrons, protons, muons) have the same essence.

> *Exchange No 2* (exchange of essences). The result of an exchange of an object A, whose essence is E_A, with an object B, whose essence is E_B, is a situation in which there is an object A' possessing properties E_A and all non-essential properties of B (relational

1. Tim Maudlin makes a similar observation in the context of the debate on the ontological status of space-time [Maudlin 1988, 83 ff.]. He points out that the fact that a diffeomorphism transforms one mathematical representation of space-time into a distinct representation does not imply that the corresponding physical realizations have to be different. Maudlin addresses a question very similar to the one we consider here: what is the best physical interpretation of the mathematical notion of a diffeomorphism connecting individual mathematical points? And his solution is closely related to the essentialist interpretation of permutation that I present later in this section.

> and non-relational), and an object B' possessing properties E_B and all non-essential properties of A.

If we consider for instance an electron in a particular quantum state $|u\rangle$ and a positron in a different state $|v\rangle$, then after their exchange-of-essences we will have a new situation in which some electron is in state $|v\rangle$, and some positron is in state $|u\rangle$.

The third interpretation requires an introduction of a new and controversial metaphysical notion. This is the notion of the *haecceity* of an object (also called *primitive thisness*). It is sometimes claimed that apart from its "ordinary" properties, essential or not, each object comes equipped with a special property, which is simply defined as being identical with itself and nothing else. Haecceity is well known to be offensive to any genuine empiricist. It cannot be characterized in a qualitative way, nor can it be directly observed or detected. And yet some philosophers feel that haecceity is necessary in order to speak about the relation of numerical distinctness and identity that is conceptually independent from qualitative identity. For now I do not wish to enter the philosophical debate on the nature and admissibility of the concept of haecceity.[2] Instead, I am simply going to introduce my third concept of exchange of objects.

> *Exchange No 3* (exchange of haecceities). The result of an exchange of an object A possessing haecceity H_A with an object B possessing haecceity H_B is a situation in which an object possessing haecceity H_A has all the properties (relational and non-relational) of B, and an object possessing H_B has all the properties of A.

It is characteristic that the process of a type 3 exchange results in a situation which is qualitatively (and hence empirically) indiscernible from the initial one, and yet we assume this situation to be ontologically different (it is supposed to be a genuinely new state of affairs). It is a scenario in which this table becomes qualitatively indistinguishable from that chair (and also assumes the location of the chair) without actually ceasing to be itself. Of course for this notion of exchange to be consistent we have to assume that objects do not possess any essential properties except their haecceities.

Finally, we may want to introduce an even thinner concept of exchange. In the exchange of the third type there was no epistemological difference between the initial and the final states, but an ontological one. Now we consider a case in which there is no ontological difference but a mere difference in language.

> *Exchange No 4* (exchange of labels). An exchange of an object A which bears a label L_A with an object B which bears a label L_B results in a situation in which there are objects A' and B'

2. An excellent philosophical discussion of the notion of haecceity in the context of quantum mechanics can be found in [Teller 1995, 16–35].

qualitatively and otherwise identical with A and B and such that A' bears the label L_B while B' bears the label L_A.

Given that the idea of an exchange of objects somehow involves the notion of retaining numerical identity in spite of undergoing superficial changes, we may note that each of the four introduced concepts of exchange corresponds to a slightly different intuition of what it takes for an object to remain the same entity. Exchange No 1 presupposes that an object's location is irrelevant to its identity, and that retaining all other properties is sufficient for it to be itself. Definition 2 implies that for an object to remain itself it is necessary that it should keep a particular subset of the set of its properties. According to the third notion, an object's identity is defined by its haecceity. The fourth option seems to be based on the rather absurd idea that the identity of an object can be somehow associated with its name.

3 Permutations in physics

In the next step we will address the question of which of the four available notions of exchange should be used as a physical interpretation of the mathematical notion of permutation. As a first example, let us consider the classical state of two particles at time t given by their position and velocity as follows:

$$r_1(t) = r, v_1(t) = v,$$
$$r_2(t) = r', v_2(t) = v'.$$

The result of the permutation of indices is the set of the following functions:

$$r_2(t) = r, v_2(t) = v,$$
$$r_1(t) = r', v_1(t) = v'.$$

From this it clearly follows that the corresponding physical exchange of two particles cannot be interpreted as exchange No 1, since in that case the resulting functions would be

$$r_1(t) = r', v_1(t) = v,$$
$$r_2(t) = r, v_2(t) = v'.$$

The idea of an exchange of position obviously does not take into account the fact that in physics position is treated as no different from any other variable characterizing a particle (velocity, momentum, angular momentum, etc.). Of the remaining three notions of a physical exchange, the exchange of labels is the least interesting because it is essentially a redescription of the same physical situation. Thus it can be claimed that there are only two interesting notions of exchange available: the exchange of essences and of haecceities.

The difference between the two concepts is clearly visible when we consider a permutation of two particles of different types. If we interpreted a permutation of indices in the description of the state of a positron and an electron as representing exchange of essences, then the permuted function would describe a situation in which the electron is in the state initially occupied by the positron, and vice versa. In contrast, the exchange of haecceities leads to the state in which the object possessing the haecceity of the electron now has all the properties (state-dependent and state-independent) of the positron, and likewise for the positron. Thus the permuted and non-permuted functions describe ontologically distinct states which are nevertheless empirically indistinguishable.

Let us now apply our selected interpretations of physical exchange to the analysis of the fundamental symmetrization/antisymmetrization postulate of quantum mechanics. The textbook way to introduce this postulate is through the concept of *exchange degeneracy*.[3] Considering the joint state of two particles of the same type such that one of them occupies state $|u\rangle$ whereas the other one is in a different state $|v\rangle$, we should observe that the two permuted states $|u\rangle_1|v\rangle_2$ and $|v\rangle_1|u\rangle_2$ are empirically indistinguishable. According to the essentialist approach this indistinguishability comes from the fact that both bi-partite states represent one and the same physical state of affairs. On the other hand, the haecceitist approach admits that there is a difference between the permuted and non-permuted states, but this difference cannot give rise to any observational effects, as haecceities are not empirically accessible.

In order to avoid the degeneracy problem, we adopt the symmetrization postulate, which narrows down the admissible states to the symmetric (occupied by bosons) and antisymmetric ones (applicable to fermions). Thus the only vector that can properly represent the above-discussed state of two electrons is the antisymmetric superposition

$$(A) \qquad \frac{1}{\sqrt{2}}(|u\rangle_1|v\rangle_2 - |v\rangle_1|u\rangle_2).$$

In the case of bosons, the state has to be symmetric

$$(S) \qquad \frac{1}{\sqrt{2}}(|u\rangle_1|v\rangle_2 + |v\rangle_1|u\rangle_2).$$

Given that in the quantum-mechanical formalism the sign of a vector has no physical meaning, it is commonly accepted that both types of vectors display the required permutation invariance which follows from the indistinguishability postulate regarding particles of the same type.

3. For an extended discussion, see e.g., [Cohen-Tannoudji, Diu *et al.* 1977, 1370–1408].

4 The Indiscernibility Thesis (IT)

The standard view is that this permutation invariance has dramatic consequences regarding the ontological status of quantum particles of the same type. Most famously, it is argued that fermions and bosons of the same type are indiscernible by their properties and relations, and hence they violate the Principle of the Identity of Indiscernibles (PII). Recent foundational work on the notion of discernibility has revealed that there are many non-equivalent ways of interpreting this concept,[4] so we have to be precise about what particular type of discernibility is claimed to be violated by quantum particles. The logically strongest grade of discernibility is known as *absolute discernibility*, and it can be roughly defined as follows: two objects a and b are absolutely discernible iff there is an open formula in one variable consisting of predicates representing admissible properties or relations which is satisfied by a but not by b. The Indiscernibility Thesis applied to quantum particles of the same type can be formulated as the negation of their absolute discernibility:

> (IT) Distinct fermions (bosons) of the same type are never absolutely discernible by their properties or relations.

Although suggestions that quantum particles may violate PII were made quite early on in the history of quantum mechanics, the first rigorous proof of this fact was given in [French & Redhead 1988] and was subsequently generalized and improved upon in other publications.[5] French & Redhead's proof is based on the assumption that when we consider a set of n particles of the same type, any property of the ith particle can be represented by an operator of the form $O_i = I^{(1)} \otimes I^{(2)} \otimes \ldots \otimes O^{(i)} \otimes \ldots \otimes I^{(n)}$, where O is a Hermitian operator acting on the single-particle Hilbert space $\mathcal{H}$. Now it is easy to prove that the expectation values of two such operators O_i and O_j calculated for symmetric and antisymmetric states are identical. Similarly, it can be proved that the probabilities of revealing any value of observables of the above type

4. See for instance a comprehensive logical analysis of various grades of discernibility in [Ladyman, Linnebo *et al.* 2012] and [Bigaj 2014]. Simon Saunders noticed that the notion of discernibility figuring in the usual formulation of PII admits different logical reconstructions. Saunders rediscovered the distinction made by Quine between absolute, relative and weak grades of discernibility, and suggested that quantum particles are indeed weakly discernible. This claim was subsequently accepted and refined in [Saunders 2006], [Muller & Saunders 2008], [Muller & Seevinck 2009]. Criticism of the weak discernibility thesis can be found, e.g., in [Hawley 2006], [Dieks & Versteegh 2008], [van Fraassen & Peschard 2008], [Ladyman & Bigaj 2010].

5. The list of publications analyzing the violation of PII in quantum mechanics is long, and it contains, among others, [van Fraassen 1991], [Butterfield 1993], [Huggett 2003], [French & Krause 2006]. IT is so commonly accepted in the literature that it could be referred to as the Received View, if not for the fact that Steven French has already appropriated this term to speak about a different position which questions the individuality of quantum particles of the same type, see e.g., [French 2011].

conditional upon any measurement outcome previously revealed are the same for all n particles.

In what follows I will argue that the Indiscernibility Thesis is actually contingent upon the selection of one of the two available interpretations of exchange of particles that we have discussed in the previous section. More specifically, I will try to show that the argument in favour of IT goes through only if we accept the exchange of haecceities interpretation. However, under the alternative essentialist interpretation, it can actually be argued that fermions and bosons are absolutely discernible in certain typical configurations.

My first argument is based on the observation that the exchange-of-essences interpretation leads to the symmetrization postulate regarding admissible observables, which effectively excludes non-symmetric observables used by French & Redhead in their proof of IT. To see that this is the case, let us recall that if we interpret the permutation P_{12} as exchanging essences of particles 1 and 2, the mathematical vectors $|\psi\rangle$ and $P_{12}|\psi\rangle$ actually represent one and the same physical state (under the condition that $|\psi\rangle$ describes a state of two indistinguishable particles having the same essences). Hence no physically meaningful observable can discriminate between the two permuted vectors. To put it more precisely, the only admissible operators are those whose expectation values are identical in "both" states $|\psi\rangle$ and $P_{12}|\psi\rangle$: $\langle\psi|O|\psi\rangle = \langle\psi|P_{12}OP_{12}|\psi\rangle$. But of course this equation must hold regardless of the choice of the state $|\psi\rangle$, and this means that the operator O commutes with the permutation operator P_{12}. Yet clearly the operators O_i introduced by French & Redhead do not commute with permutation operators, as can be seen in the following commutation relation: $P_{ij}O_iP_{ij} = O_j$.

French & Redhead are aware of the problem [French & Redhead 1988, 239]. Their response to it is based on the distinction between observable and unobservable properties. But I believe that this reply has no force in the context of the exchange-of-essences interpretation. The permuted states $|\psi\rangle$ and $P_{12}|\psi\rangle$ are not merely observationally indistinguishable—they are metaphysically identical. An operator which "sees" a difference between numerically identical physical states merely because of their (or rather "its") different mathematical representations cannot possibly represent any physically meaningful property, whether observable or not.

In my mind French & Redhead's response makes sense only under the alternative haecceity interpretation. Here the vectors $|\psi\rangle$ and $P_{12}|\psi\rangle$ represent observationally indistinguishable but numerically distinct states of affairs. Thus, it can be claimed that operators O_i and O_j, whose expectation values in states $|\psi\rangle$ and $P_{12}|\psi\rangle$ are different, represent some "hidden" properties of the entire system, reflecting the ontological distinctness between the permuted states. Speaking loosely, each operator O_i is "attached" to a different haecceity via its label i, so when we swap haecceities between particles i and j, clearly the result should be "registered" by O_i. But no physically meaningful operator can register any difference between a situation and itself, regardless of how we decide to represent it mathematically.[6]

5 Essentialism and discernibility

But this is not the end of the story. The fact that one particular argument in favor of IT turns out to be incorrect does not show that the thesis itself is false. We need a direct proof that the exchange-of-essences interpretation leads to the conclusion that some particles of the same type are indeed absolutely discernible. Finding just one physical property such that *only* one particle occupying a joint symmetric/antisymmetric state possesses it is all we need to reach our goal. But before we can do that, we must address the question of how to properly represent measurable characteristics of individual particles when the admissible operators are restricted to symmetric ones.

Let us consider any one-dimensional projection operator P acting on a single-particle Hilbert space. As is well-known, such an operator is taken to represent a specific quantum-mechanical property of an individual particle. Moreover, using the whole family of such projection operators we can in principle describe any physical property of a particle. But our goal now is to construct a new projector acting on the tensor product of two Hilbert spaces which could represent the statement that (at least) one of two indistinguishable particles possesses property P. It is relatively straightforward to notice that such an operator Ω should satisfy the following desiderata:

(1) Ω should be Hermitian,

(2) Ω should be symmetric,

(3) Ω should be a projector (and therefore idempotent),

(4) Ω should be the sum of tensor products of one-particle operators involving only P and I (the identity).

From conditions (2) and (4) it follows that the most general form Ω can have is the following:

$$\Omega = aP \otimes I + aI \otimes P + bP \otimes P.$$

6. Nick Huggett generalizes French & Redhead's proof of IT in a way which may seem to threaten my argument [Huggett 2003]. He shows that we don't actually need to assume that observables O_i have the specific non-symmetric form required by French & Redhead. Huggett's proof of the fact that O_i and O_j have the same expectation values in symmetric (antisymmetric) states relies on two assumptions only: the commutation relation $P_{ij}O_iP_{ij} = O_j$ (which he calls the conjugacy condition), and the independence condition $P_{ij}O_kP_{ij} = O_k$ (for $k \neq i$ and j). However, I'd like to point out that if we assume (as is required under the essentialist view) that operators O_i are symmetric, the conjugacy condition immediately implies that $O_i = O_j$ for all i and j. It is hardly an exciting theorem that identical operators have the same expectation values in all states.

Given that Ω is assumed to be Hermitian, coefficients a and b have to be real. Now we can apply requirement (3):

$$\Omega^2 = \Omega.$$

Let us calculate the square of Ω (using the fact that $P^2 = P$):

$$\Omega^2 = a^2 P \otimes I + a^2 I \otimes P + (2a^2 + 4ab + b^2) P \otimes P.$$

Comparing formulas for Ω and Ω^2 we can first derive $a^2 = a$. This equation obviously has two solutions in real numbers (0 and 1), but we can discard the value 0, as the operator $P \otimes P$ clearly represents the situation in which both particles have the same property. If we put $a = 1$, we can easily solve the quadratic equation in b which arises as the result of equating the coefficients of the component $P \otimes P$ in the expansions of Ω and Ω^2. Thus the only two solutions are as follows:

$$\Omega_1 = P \otimes I + I \otimes P - P \otimes P,$$
$$\Omega_2 = P \otimes I + I \otimes P - 2P \otimes P.$$

However, we don't have to make a choice between Ω_1 and Ω_2 in order to prove the following theorem. It is not difficult to observe that Ω_1 represents the question "Does at least one particle possess property P?" while Ω_2 the question "Is it true that one particle possesses property P while the other does not possess P?" See an extensive analysis given in [Ghirardi, Marinatto *et al.* 2002], [Ghirardi & Marinatto 2004].

> **Theorem 1.** Let Ψ be a normalized vector $a|u\rangle_1|v\rangle_2 + b|v\rangle_1|u\rangle_2$ where $|u\rangle$ and $|v\rangle$ are mutually orthogonal unit vectors, and let $P = |u\rangle\langle u|$. Then the expectation value of both operators Ω_1 and Ω_2 in state Ψ is 1.

Here is a sketch of the calculation confirming this fact:

$$\langle\Psi|P\otimes I|\Psi\rangle = \langle a^* uv + b^* vu|P\otimes I|auv + bvu\rangle = a^* a\langle u|P|u\rangle + b^* b\langle v|P|v\rangle = |a|^2.$$

Analogously, it can be showed that

$$\langle\Psi|I \otimes P|\Psi\rangle = |b|^2.$$

And because the expectation value of $P \otimes P$ in Ψ vanishes due to the orthogonality relation between $|u\rangle$ and $|v\rangle$, we finally arrive at the sought-after result:

$$\langle\Psi|\Omega_1|\Psi\rangle = \langle\Psi|\Omega_2|\Psi\rangle = |a|^2 + |b|^2 = 1$$

What is the meaning of this formal derivation? It can be unpacked as follows: given the only available mathematical representation of the statement "At least one particle possesses property P", if the system is prepared in a superposition of the product of two orthogonal states $|u\rangle_1|v\rangle_2$ and its permuted form $|v\rangle_1|u\rangle_2$, at least one of the two particles possesses the property associated with state $|u\rangle$. But exactly the same can be proved with respect to the state $|v\rangle$. Consequently, we have to admit that at least one particle has the property associated with $|u\rangle$, and at least one particle has the property associated with $|v\rangle$. But clearly one particle cannot be both in state $|u\rangle$ and state $|v\rangle$. Thus we have proved that the particles prepared in state Ψ are discernible by their properties $P = |u\rangle\langle u|$ and $Q = |v\rangle\langle v|$. This result obviously applies to bosons and fermions, as symmetric and antisymmetric states are just special cases of the superposition Ψ.

It may be instructive to see why this conclusion is avoidable under the alternative, haecceistic interpretation of permutation. Of course, the formal result of Theorem 1 still stands, as it is a mathematical fact, but its physical interpretation changes. Haecceitism implies that there is a meaningful difference between labels, as they refer to numerically distinct entities with different primitive identities. Consequently, we can conceptually (although not observationally) distinguish between statements "Particle 1 has property P" and "Particle 2 has property P". The first statement is deemed true if and only if the operator $P \otimes I$ receives expectation value 1 in state Ψ, whereas the second one corresponds to the operator $I \otimes P$ and its expectation value. For the haecceitist the correct interpretation of the statement "At least one particle possesses property P" is just the classical disjunction of the above-mentioned individual statements: "Particle 1 has property P or particle 2 has property P". And in the case in which none of the disjuncts receives the value "true" the entire disjunction cannot be true.

The haecceitist interprets the fact that the operator Ω_1 has its expectation value equal 1 in Ψ as a mere indication of the fact that when we decide to measure P on both particles, one measurement will reveal value 1 with certainty. But this doesn't mean that any particle possesses the corresponding property P before the measurement. So Ω_1 can be construed as referring to whatever property of the entire system is responsible for the predicted behavior (most likely this property has a fundamental dispositional character). But I would like to stress that without the "thick" metaphysics of primitive identities the disjunctive interpretation of the statement "At least one particle has property P" would not be available. Without haecceities we have only two options: either to accept Ω_1 as a formal representation of this property, or to admit that the property in question is not expressible at all in our impoverished symmetric language.

6 Conclusion

I have laid down two main philosophical positions regarding the meaning of permutation invariance in quantum mechanics: the essentialist view and the haecceitistic view. I have argued that both views come in whole packages, including a lot more than the mere philosophical interpretations of the notion of permutation. Essentialism leads to the strong symmetrization postulate with respect to admissible observables, which prevents us from representing properties of individual particles with the help of non-symmetric, label-bearing operators. Generally, essentialism repudiates the use of labels (indices) as names with fixed reference, and instead treats them only as formal devices enabling us to consider certain mathematical symmetries. Finally, it can be argued that insofar as essentialism is capable of formulating and solving the problem of absolute discernibility of particles of the same type at all, its answer is that both fermions and bosons can be actually discerned by their properties.

On the other side of the divide, haecceitism has to make the distinction between physically meaningful Hermitian operators and operators corresponding to observable properties. Operators which represent properties of individual particles are meaningful but, strangely enough, they are not literally *observables*. Labels used in the formal description of many-particle states are to be treated literally: they follow the primitive identity of individual objects. The Indiscernibility Thesis follows under this view from the fact that the expectation values for all single-particle operators are the same in antisymmetric/symmetric states. One surprising feature of haecceitism is that it is actually a necessary component of IT. Without haecceitism the argument for the indiscernibility claim could not even get off the ground.[7]

Due to the lack of space I can't discuss in detail what I consider the greatest challenge to the essentialist interpretation and the associated claim that absolute discernibility is attainable for quantum particles of the same type. This challenge is a consequence of the fact that in the state resulting from the antisymmetrization of the product state $|u\rangle_1|v\rangle_2$ there are infinitely many projectors representing single-particle properties other than $|u\rangle$ and $|v\rangle$ whose expectation values equal 1. As a result, it seems that we would have to admit that individual particles possess mutually incompatible properties. This

7. This point, as I believe, must have somehow escaped the notice of main experts in the field. For instance, French & Krause famously defend their thesis of the underdetermination of metaphysics by physics by pointing out that the quantum-mechanical description of systems of many particles can support two different metaphysical views: one stating that particles are individuals distinguished by their unique haecceities, and the other that they are non-individuals [French & Krause 2006, 189–197]. But if quantum particles are non-individuals which cannot be meaningfully referred to with the help of different labels, then as I noticed the standard argument in favor of the Indiscernibility Thesis cannot even be formulated. On the other hand, it may seem that the essentialist interpretation of exchange will be more sympathetic to the "particles as non-individuals" view. But this suggestion flies in the face of the fact proved above that under this interpretation particles can be discerned by their qualitative properties. Thus it looks like the non-individual view is excluded by both interpretations of exchange considered in this article. One possibility of making room for this position is to argue that under the non-individual view the notion of an exchange of particles is meaningless, and as such does not require any physical interpretation whatsoever.

problem requires an extensive and thorough evaluation which has to be saved for another occasion.

I would like to end this survey with a plea on behalf of essentialism. The fact that the essentialist approach admits the possibility of discerning quantum particles by their properties fits well the everyday practice of experimental physicists who have no qualms about talking of the electron in a bubble chamber as being an entity different from an electron in the Andromeda galaxy. Although essentialism entails that in some cases quantum particles may occupy states which render them indiscernible, this does not necessarily rob them of the status of individuals, if we follow Dieks & Versteegh and define individuals as objects for which it is possible to be in a state in which they possess different properties [Dieks & Versteegh 2008]. I am aware of the conceptual difficulties afflicting this position, and I admit that at this point I can't offer a satisfactory solution to all of them. But I believe that the advantages of the essentialist interpretation merit further investigation into this new approach to the problem of identity and individuality in quantum mechanics.

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